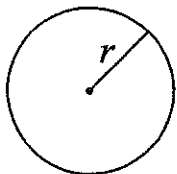


Squares, Square Roots, Cubes and Cube Roots of Some numbers

<u>Number</u>	<u>Number Squared</u>	<u>Number Cubed</u>
0	0	0
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000
11	121	1331
12	144	1728
13	169	2197
14	196	2744
15	225	3375
16	256	4096
17	289	4913
18	324	5832
19	361	6859
20	400	8000
21	441	9261
22	484	10648
23	529	12167
24	576	13824
25	625	15625
26	676	17576
27	729	19683
28	784	21952
29	841	24389
30	900	27000

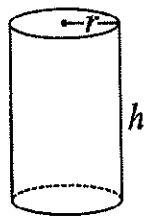
Use the information below to answer questions on the Mathematics test.

Circle



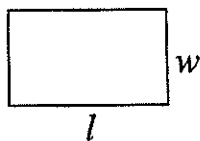
$\pi \approx 3.14$
 Area = πr^2
 Circumference = $2\pi r$

Cylinder



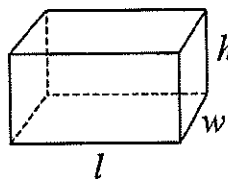
Volume = $\pi r^2 h$
 Surface Area = $2\pi r^2 + 2\pi r h$

Rectangle



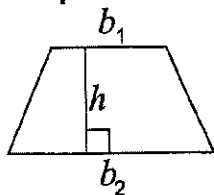
Area = lw
 Perimeter = $2l + 2w$

Rectangular Solid



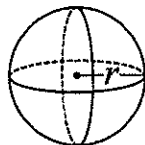
Volume = lwh
 Surface Area = $2wl + 2lh + 2wh$

Trapezoid



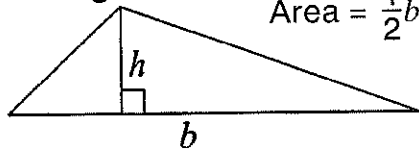
Area = $\frac{1}{2}h(b_1 + b_2)$

Sphere



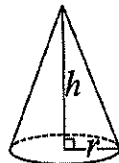
Volume = $\frac{4}{3}\pi r^3$

Triangle



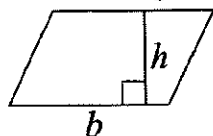
Area = $\frac{1}{2}bh$

Cone



Volume = $\frac{1}{3}\pi r^2 h$

Parallelogram



Area = bh

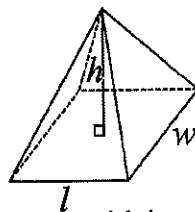
Metric Units of Length

- 1 kilometer = 1,000 meters
- 1 centimeter = 0.01 meter
- 1 millimeter = 0.001 meter
- 1 micrometer = 0.000001 meter

U.S. Unit Conversions

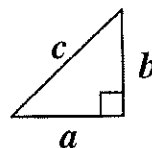
- 8 fluid ounces = 1 cup
- 2 cups = 1 pint
- 2 pints = 1 quart
- 4 quarts = 1 gallon
- 16 ounces = 1 pound
- 5,280 feet = 1 mile

Rectangular Pyramid



Volume = $\frac{1}{3}lwh$

Pythagorean Theorem



$a^2 + b^2 = c^2$

Cartesian Distance Formula

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 (see note below)

Slope Formula

slope = $\frac{y_2 - y_1}{x_2 - x_1}$

(see note below)

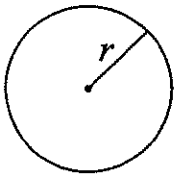
NOTE: Point A: (x_1, y_1)

Point B: (x_2, y_2)

Mathematics Reference Sheet

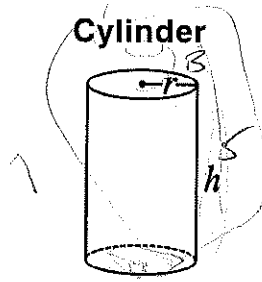
Use the information below to answer questions on the Mathematics test.

Circle



$\pi \approx 3.14$
 Area = πr^2
 Circumference = $2\pi r$

Cylinder



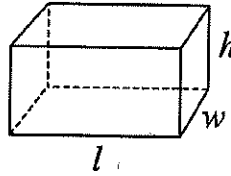
Volume = $\pi r^2 h$
 Surface Area = $2\pi r^2 + 2\pi r h$

Rectangle



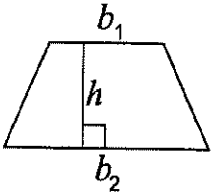
Area = lw
 Perimeter = $2l + 2w$

Rectangular Solid



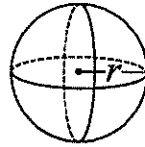
Volume = lwh
 Surface Area = $2wl + 2lh + 2wh$

Trapezoid



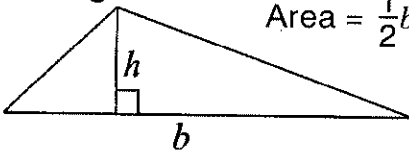
Area = $\frac{1}{2}h(b_1 + b_2)$

Sphere



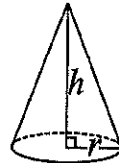
Volume = $\frac{4}{3}\pi r^3$

Triangle



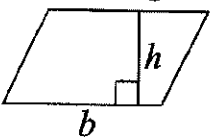
Area = $\frac{1}{2}bh$

Cone



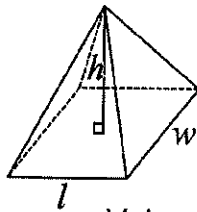
Volume = $\frac{1}{3}\pi r^2 h$

Parallelogram



Area = bh

Rectangular Pyramid



Volume = $\frac{1}{3}lwh$

Cartesian Distance Formula

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(see note below)

Slope Formula

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

(see note below)

NOTE: Point A: (x_1, y_1)
Point B: (x_2, y_2)

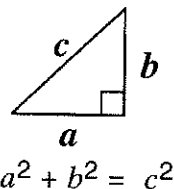
Metric Units of Length

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- 1 millimeter = 0.001 meter
- 1 micrometer = 0.000001 meter

U.S. Unit Conversions

- 8 fluid ounces = 1 cup
- 2 cups = 1 pint
- 2 pints = 1 quart
- 4 quarts = 1 gallon
- 16 ounces = 1 pound
- 5,280 feet = 1 mile

Pythagorean Theorem



Right Triangle Trigonometry

You may have been introduced to Trigonometry in **Geometry**, when you had to find either a **side length** or **angle measurement** of a triangle. Trigonometry is basically the study of **triangles**, and was first used to help in the computations of astronomy. Today it is used in engineering, architecture, medicine, physics, among other disciplines.

The **6 basic trigonometric functions** that you'll be working with are **sine** (rhymes with "sign"), **cosine**, **tangent**, **cosecant**, **secant**, and **cotangent**. (Don't let the fancy names scare you; they really aren't that bad).

With **Right Triangle Trigonometry**, we use the trig functions on **angles**, and get a **number** back that we can use to get a side measurement, as an example. Sometimes we have to work backwards to get the **angle measurement back** so we have to use what we call an **inverse trig function**. **But basically remember that we need the trig functions so we can determine the sides and angles of a triangle that we don't otherwise know.**

Later, we'll see how to use trig to find areas of triangles, too, among other things.

You may have been taught **SOH – CAH – TOA (SOHCAHTOA)** (pronounced "so – kuh – toe – uh") to remember these. Back in the old days when I was in high school, we didn't have **SOHCAHTOA**, nor did we have fancy calculators to get the values; we had to look up trigonometric values in tables.

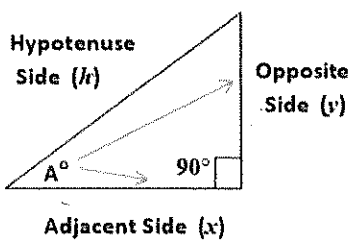
Remember that the definitions below assume that the triangles are **right triangles**, meaning that they all have **one right angle (90°)**. Also note that in the following examples, our angle measurements are in **degrees**; later we'll learn about another angle measurement unit, **radians**, which we'll discuss [here in the Angles and Unit Circle section](#).

Basic Trigonometric Functions (SOH – CAH – TOA)

Here are the **6 trigonometric functions**, shown with both the SOHCAHTOA and **Coordinate System Methods**.

Note that the second set of three trig functions are just the reciprocals of the first three; this makes it a little easier! Note that the **cosecant (csc)**, **secant (sec)**, and **cotangent (cot)** functions are called **reciprocal functions**, or **reciprocal trig functions**, since they are the reciprocals of **sin**, **cos**, and **tan**, respectively.

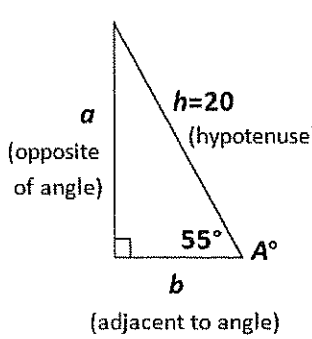
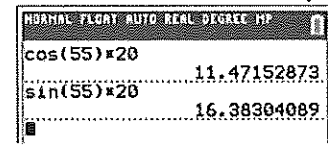
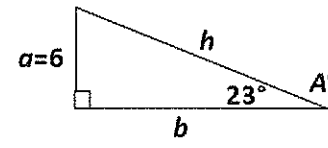
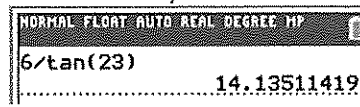
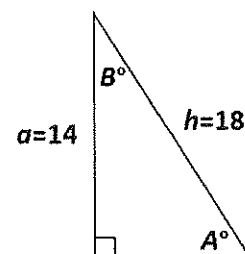
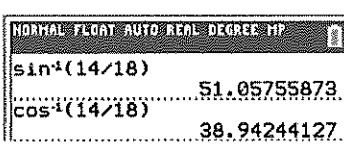
For the coordinate system method, assume that the vertex of the angle in the triangle is at the origin (0,0):

Right Triangle	SOH-CAH-TOA Method	Coordinate System Method
	<p>SOH: Sine (A) = $\sin(A) = \frac{\text{Opposite}}{\text{Hypotenuse}}$</p> <p>CAH: Cosine (A) = $\cos(A) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$</p> <p>TOA: Tangent (A) = $\tan(A) = \frac{\text{Opposite}}{\text{Adjacent}}$</p>	<p>$\sin(A) = \frac{y}{h}$</p> <p>$\cos(A) = \frac{x}{h}$</p> <p>$\tan(A) = \frac{y}{x}$</p>
	<p>cosecant (A) = $\csc(A) = \frac{1}{\sin(A)} = \frac{\text{Hypotenuse}}{\text{Opposite}}$</p> <p>secant (A) = $\sec(A) = \frac{1}{\cos(A)} = \frac{\text{Hypotenuse}}{\text{Adjacent}}$</p> <p>cotangent (A) = $\cot(A) = \frac{1}{\tan(A)} = \frac{\text{Adjacent}}{\text{Opposite}}$</p>	<p>$\csc(A) = \frac{1}{\sin(A)} = \frac{h}{y}$</p> <p>$\sec(A) = \frac{1}{\cos(A)} = \frac{h}{x}$</p> <p>$\cot(A) = \frac{1}{\tan(A)} = \frac{x}{y}$</p>

Here are some **example problems**. Note that we commonly use **capital letters** to represent angle measurements, and the same letters in **lower case** to represent the side measurements **opposite those angles**.

We also use the **theta symbol θ** to represent angle measurements, as we'll see later. Note also in these problems, we need to put our calculator in the **DEGREE** mode.

And don't forget the **Pythagorean Theorem** ($a^2+b^2=c^2$, where a and b are the "legs" of the triangle, and c is the hypotenuse), and the fact that the sum of all angles in a triangle is **180°**.

Problem	Explanation	Calculator Steps/Checking
<p>Find the value of a and b:</p>  <p>a (opposite of angle) $h=20$ (hypotenuse) 55° A° b (adjacent to angle)</p>	<p>To get side a, we need to use</p> $\sin(A) = \frac{\text{Opposite}}{\text{Hypotenuse}}, \text{ where } A \text{ is } 55^\circ:$ $\sin(55^\circ) = \frac{a}{20}; \text{ cross multiply}$ $a = \sin(55^\circ) \cdot 20 = 16.383$ <hr/> <p>To get side b, we need to use</p> $\cos(A) = \frac{\text{Adjacent}}{\text{Hypotenuse}}, \text{ where } A \text{ is } 55^\circ:$ $\cos(55^\circ) = \frac{b}{20}; \text{ cross multiply}$ $b = \cos(55^\circ) \cdot 20 = 11.472$	<p>Hit COS MODE and scroll down and to the right to make sure you're in DEGREE mode.</p> <p>Then use the COS and SIN keys:</p>  <p>Let's check our sides using the Pythagorean Theorem:</p> $a^2 + b^2 = c^2$ $(16.383)^2 + (11.471)^2 = 399.99 \approx (20)^2$
<p>Find the value of b:</p>  <p>$a=6$ h 23° A° b</p>	<p>To get side b, we need to use</p> $\tan(A) = \frac{\text{Opposite}}{\text{Adjacent}}, \text{ where } A \text{ is } 23^\circ:$ $\tan(23^\circ) = \frac{6}{b}; \text{ cross multiply to get}$ $\tan(23^\circ) \cdot b = 6$ <p>or turn proportion sideways with an = sign:</p> $\frac{b}{1} = \frac{6}{\tan(23^\circ)}; a = 14.135$	<p>Use the TAN key:</p>  <p>If we needed to also find h, we could either use $\sin(23^\circ) = \frac{6}{h}$ or Pythagorean Theorem; both ways reveal that $h = 15.356$.</p>
<p>Find the values of A and B:</p>  <p>$a=14$ B° $h=18$ A°</p>	<p>This one's a little trickier since we need to find angle measurements instead of side measurements; we'll need to use the $\sin^{-1}(A)$ and $\cos^{-1}(A)$ (2nd sin and 2nd cos on the calculator) to get the angles back.</p> <p>For angle A, we can use sin, since we have the opposite side (14) and hypotenuse (18):</p> $\sin(A) = \frac{\text{Opp}}{\text{Hyp}} = \frac{14}{18}; A = \sin^{-1}\left(\frac{14}{18}\right) = 51.1^\circ$ <p>And for angle B we use cos:</p> $\cos(B) = \frac{\text{Adj}}{\text{Hyp}} = \frac{14}{18}; B = \cos^{-1}\left(\frac{14}{18}\right) = 38.9^\circ$	<p>Use the SIN and COS keys:</p>  <p>Let's check to make sure the sum of all angles is 180:</p> $51.1^\circ + 38.9^\circ + 90^\circ \text{ (right angle)} = 180^\circ.$

Trigonometry Word Problems

Here are some types of **word problems** that you might see when studying right angle trigonometry.

Note that the **angle of elevation** is the angle up from the ground; for example, if you look up at something, this angle is the angle between the ground and your line of site.

The **angle of depression** is the angle that comes down from a straight horizontal line in the sky. (For example, if you look down on something, this angle is the angle between your looking straight and your looking down to the ground). For the **angle of depression**, you can typically use the fact that **alternate interior angles of parallel lines are congruent** (sorry, too much Geometry!) to put that angle in the triangle on the ground (we'll see examples).

Note that **shadows** in these types of problems are typically **on the ground**. When the sun casts the shadow, the **angle of depression** is the same as the **angle of elevation** from the ground up to the top of the object whose shadow is on the ground.

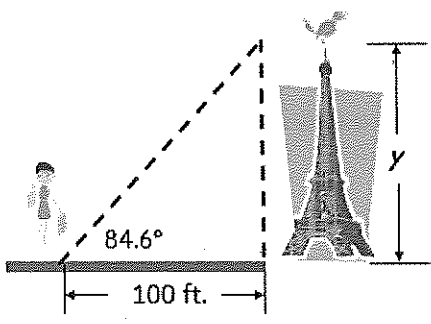
Also, the **grade** of something (like a road) is the **tangent** (rise over run) of that angle coming from the ground. Usually the grade is expressed as a percentage, and you'll have to convert the percentage to a decimal to use in the problem.

And, as always, always draw pictures!

Angle of Elevation Problem:

Devon is standing 100 feet from the Eiffel Tower and sees a bird land on the top of the tower (she has really good eyes!). If the **angle of elevation** from Devon to the top of the Eiffel Tower is close to 84.6° , how tall is the tower?

Solution: This is a good example how we might use trig to get distances that are typically difficult to measure. Note that the **angle of elevation comes up off of the ground**.

Picture	Math
	<p>To get the height y, we need to use</p> $\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}}, \text{ where } \theta \text{ is } 84.6^\circ:$ $\tan(84.6^\circ) = \frac{y}{100}; \text{ cross multiply to get}$ $y = \tan(84.6^\circ) \cdot 100 \approx 1058$ <p>So the Eiffel Tower is roughly 1058 feet tall.</p>

<

Angle of Depression Problem:

From the top of a building that is **200 feet** tall, Meryl sees a car coming towards the building. (Somehow she knows that) the **angle of depression** when she first saw the car was 20° and when she stopped looking at it was 40° degrees. How far did the car travel?

Solution: The first step is to draw a picture, and note that we can sort of "reflect" the angles of depression down to angles of elevation, since the horizon and ground are parallel. Then we get to use trig!

Picture	Math
	<p>The trick is to see that we can get distances y and z using the tangent function, and we need to subtract the two distances to get x, the distance the car travels.</p> <p>To get y: $\tan(40^\circ) = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{200}{y}$; $y = \frac{200}{\tan(40^\circ)} \approx 238.4$</p> <p>To get z: $\tan(20^\circ) = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{200}{z}$; $z = \frac{200}{\tan(20^\circ)} \approx 549.5$</p> <p>To get x, we subtract y from z, so the car moved $549.5 - 238.4 = 311.1$ feet while Meryl was watching it.</p>

Right Triangle Systems Problem:

Here's a problem where it's easiest to solve it using a System of Equations:

Two girls are standing **100 feet** apart. They both see a beautiful seagull in the air between them. The angles of elevation from the girls to the bird are **20°** and **45°**, respectively. How high up is the seagull?

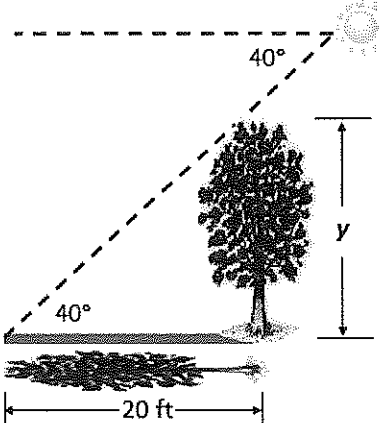
Picture	Math
	<p>The trick here is divide up the 100 ft into x and the other is $100 - x$ (put in real numbers to see how we get this) since we have two triangles. Then we will have two equations (one for each triangle) and two unknowns:</p> $\tan(20^\circ) = \frac{y}{x}; \quad \tan(45^\circ) = \frac{y}{100 - x}$ <p>From the first equation, we get y in terms of x: $y = \tan(20^\circ) \cdot x \approx .36397x$.</p> <p>We can plug this into the second equation to get $\tan(45^\circ) = \frac{.36397x}{100 - x}$.</p> <p>Solving for x, we get $1 = \frac{.36397x}{100 - x}$; $100 - x = .36397x$; $x \approx 73.315$. (I left more decimal places, so the final answer will be more accurate).</p> <p>Now we have to get y to find the height of the seagull; $y = .36397(73.315) = 26.684$. So the seagull is about 26.68 feet high. This was a tricky one!</p>

Solution:

Trig Shadow Problem:

The length of a tree's shadow is **20 feet** when the **angle of elevation** to the sun is **40°**. How tall is the tree?

Solution: Again, note that **shadows** in these types of problems are **on the ground**. When the sun casts the shadow, the **angle of depression** is the same as the **angle of elevation** from the ground up to the top of the tree. So let's solve using trig:

Picture	Math
	<p>To get the height y, we need to use</p> $\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}}, \text{ where } \theta \text{ is } 40^\circ:$ $\tan(40^\circ) = \frac{x}{20}$ <p>Cross multiply to get $x = \tan(40^\circ) \cdot 20 \approx 16.78$.</p> <p>So the height of the tree is approximately 16.78 feet tall. Not too bad!</p>

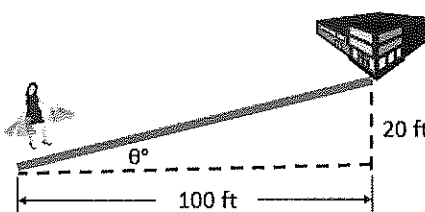
Trig Grade Problem:

Chelsea walked up a road that has a **20% grade** (she could feel it!) to get to her favorite store. At what angle does the road come up from the ground (at what angle is the road inclined from the ground)?

Solution: Remember that the grade of a road can be thought of as riserun and you usually see it as a percentage.

So a **20% grade** is the same as a grade of

20/100; for every **20 feet** the road goes up vertically, it goes **100 feet** horizontally.

Picture	Math
 <p>Grade = $\frac{20}{100} = 20\%$.</p>	<p>Since we need to find an angle measurement and we have the adjacent and opposite sides, we'll need to use the $\tan^{-1}(\theta)$ (2nd tan on the calculator and make sure it's in DEGREE mode) to get the angle back:</p> $\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{20}{100}; \quad \theta = \tan^{-1}\left(\frac{20}{100}\right) = 11.31^\circ$ <p>So the road comes up at an angle of roughly 11.31° from the ground.</p> <p>Note that if we wanted to know how long the actual slanted road is, we could just use Pythagorean Theorem, or sin or cos:</p> $\sin(11.31^\circ) = \frac{20}{x}; \quad x = \frac{20}{\sin(11.31^\circ)} \approx 101.98 \text{ ft.}$ <p>This makes sense since the grade is relatively small (note that the picture is not drawn to scale!)</p>

Understand these problems, and practice, practice, practice!

Algebra Cheat Sheet

Basic Properties & Facts

Properties of Inequalities

- If $a < b$ then $a + c < b + c$ and $a - c < b - c$
- If $a < b$ and $c > 0$ then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$
- If $a < b$ and $c < 0$ then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

Properties of Absolute Value

- $|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$
- $|a| \geq 0$
- $|ab| = |a||b|$
- $\frac{|a|}{|b|} = \frac{|a|}{|b|}$
- Triangle Inequality: $|a + b| \leq |a| + |b|$

Distance Formula

If $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ are two points the distance between them is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Complex Numbers

- $i = \sqrt{-1} \quad i^2 = -1 \quad \sqrt{-a} = i\sqrt{a}, a \geq 0$
- $(a + bi) + (c + di) = a + c + (b + d)i$
- $(a + bi) - (c + di) = a - c + (b - d)i$
- $(a + bi)(c + di) = ac - bd + (ad + bc)i$
- $(a + bi)(a - bi) = a^2 + b^2$
- Complex Modulus: $|a + bi| = \sqrt{a^2 + b^2}$
- Complex Conjugate: $\overline{a + bi} = a - bi$
- $\overline{\overline{a + bi}} = a + bi$

Properties of Radicals

- $\sqrt[n]{a} = a^{\frac{1}{n}}$
- $\sqrt[n]{a} \sqrt[n]{b} = \sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$
- $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$
- $\sqrt[n]{a^n} = a$, if n is odd
- $\sqrt[n]{a^n} = |a|$, if n is even

Arithmetic Operations

$$ab + ac = a(b + c)$$

$$\left(\frac{a}{b}\right) \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$$

$$\frac{a - b}{c - d} = \frac{b - a}{d - c}$$

$$\frac{ab + ac}{a} = b + c, a \neq 0$$

Exponent Properties

- $a^m a^n = a^{m+n}$
- $(a^n)^m = a^{nm}$
- $(ab)^n = a^n b^n$
- $a^{-n} = \frac{1}{a^n}$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- $\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}$
- $a^0 = 1, a \neq 0$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- $\frac{1}{a^{-n}} = a^n$
- $a^{\frac{1}{n}} = (a^n)^{\frac{1}{n}} = (a^{\frac{1}{n}})^n$

Properties of Radicals

- $\sqrt[n]{a} = a^{\frac{1}{n}}$
- $\sqrt[n]{a} \sqrt[n]{b} = \sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$
- $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$
- $\sqrt[n]{a^n} = a$, if n is odd
- $\sqrt[n]{a^n} = |a|$, if n is even

Logarithms and Log Properties

Definition

$y = \log_b x$ is equivalent to $x = b^y$

Example

$\log_3 125 = 3$ because $3^3 = 125$

Special Logarithms

$\ln x = \log_e x$ natural log
 $\log x = \log_{10} x$ common log
 where $e = 2.718281828\dots$

Factoring and Solving Quadratic Formula

Solve $ax^2 + bx + c = 0, a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- If $b^2 - 4ac > 0$ - Two real unequal solns.
- If $b^2 - 4ac = 0$ - Repeated real solution.
- If $b^2 - 4ac < 0$ - Two complex solutions.

Square Root Property

If $x^2 = p$ then $x = \pm\sqrt{p}$

Absolute Value Equations/Inequalities

If b is a positive number

- $|p| = b \Rightarrow p = -b$ or $p = b$
- $|p| < b \Rightarrow -b < p < b$
- $|p| > b \Rightarrow p < -b$ or $p > b$

Completing the Square

- Factor the left side

$$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$$
- Use Square Root Property

$$x - \frac{3}{2} = \pm\sqrt{\frac{29}{4}} = \pm\frac{\sqrt{29}}{2}$$
- Solve for x

$$x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$$

Solve $2x^2 - 6x - 10 = 0$

- Divide by the coefficient of the x^2
 $x^2 - 3x - 5 = 0$
- Move the constant to the other side
 $x^2 - 3x = 5$
- Take half the coefficient of x , square it and add it to both sides

$$x^2 - 3x + \left(\frac{3}{2}\right)^2 = 5 + \left(\frac{3}{2}\right)^2 = 5 + \frac{9}{4} = \frac{29}{4}$$

Functions and Graphs

Constant Function

$$y = a \text{ or } f(x) = a$$

Graph is a horizontal line passing through the point $(0, a)$.

Line/Linear Function

$$y = mx + b \text{ or } f(x) = mx + b$$

Graph is a line with point $(0, b)$ and slope m .

Slope

Slope of the line containing the two points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

Slope - intercept form

The equation of the line with slope m and y -intercept $(0, b)$ is

$$y = mx + b$$

Point - Slope form

The equation of the line with slope m and passing through the point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

Parabola/Quadratic Function

$$y = a(x - h)^2 + k \quad f(x) = a(x - h)^2 + k$$

The graph is a parabola that opens up if $a > 0$ or down if $a < 0$ and has a vertex at (h, k) .

Parabola/Quadratic Function

$$y = ax^2 + bx + c \quad f(x) = ax^2 + bx + c$$

The graph is a parabola that opens up if $a > 0$ or down if $a < 0$ and has a vertex

$$\text{at } \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right) \right).$$

Parabola/Quadratic Function

$$x = ay^2 + by + c \quad g(y) = ay^2 + by + c$$

The graph is a parabola that opens right if $a > 0$ or left if $a < 0$ and has a vertex

$$\text{at } \left(g\left(-\frac{b}{2a}\right), -\frac{b}{2a} \right).$$

Circle

$$(x - h)^2 + (y - k)^2 = r^2$$

Graph is a circle with radius r and center (h, k) .

Ellipse

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

Graph is an ellipse with center (h, k) with vertices a units right/left from the center and vertices b units up/down from the center.

Hyperbola

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

Graph is a hyperbola that opens left and right, has a center at (h, k) , vertices a units left/right of center and asymptotes that pass through center with slope $\pm \frac{b}{a}$.

Hyperbola

$$\frac{(y - k)^2}{b^2} - \frac{(x - h)^2}{a^2} = 1$$

Graph is a hyperbola that opens up and down, has a center at (h, k) , vertices b units up/down from the center and asymptotes that pass through center with slope $\pm \frac{b}{a}$.

Common Algebraic Errors

Error

$$\frac{2}{0} \neq 0 \text{ and } \frac{2}{0} \neq 2$$

Division by zero is undefined!

$$-3^2 \neq 9$$

$(-3)^2 = 9$, $(-3)^2 = 9$ Watch parenthesis!

$$(x^2)^3 = x^2 \cdot x^2 \cdot x^2 = x^6$$

$$\frac{1}{\frac{1}{2}} = \frac{1}{1+1} = \frac{1}{2}$$

A more complex version of the previous error.

$$\frac{a + bx}{a} = \frac{a}{a} + \frac{bx}{a} = 1 + \frac{bx}{a}$$

Beware of incorrect canceling!

$$-a(x - 1) = -ax + a$$

Make sure you distribute the $-$!

$$(x + a)^2 = (x + a)(x + a) = x^2 + 2ax + a^2$$

$$5 = \sqrt{25} = \sqrt{3^2 + 4^2} \neq \sqrt{3^2} + \sqrt{4^2} = 3 + 4 = 7$$

See previous error.

More general versions of previous three errors.

$$2(x + 1)^2 = 2(x^2 + 2x + 1) = 2x^2 + 4x + 2$$

$$(2x + 2)^2 = 4x^2 + 8x + 4$$

Square first then distribute!

See the previous example. You can not factor out a constant if there is a power on the parenthesis!

$$\sqrt{-x^2 + a^2} = (-x^2 + a^2)^{\frac{1}{2}}$$

Now see the previous error.

$$\frac{\frac{a}{\left(\frac{b}{c}\right)}}{\left(\frac{c}{b}\right)} = \frac{\frac{a}{1}}{\frac{b}{c}} = \frac{a}{1} \cdot \frac{c}{b} = \frac{ac}{b}$$

$$\frac{\left(\frac{a}{b}\right)}{c} = \frac{\frac{a}{b}}{\frac{c}{1}} = \frac{a}{b} \cdot \frac{1}{c} = \frac{a}{bc}$$

Algebra Review

PEARSON
Education

Numbers

FRACTIONS

■ Addition and Subtraction

- To add or subtract fractions with the same denominator, add or subtract the numerators and keep the same denominator.
- To add or subtract fractions with different denominators, find the LCD and write each fraction with this LCD. Then follow the procedure in step i.

■ Multiplication

Multiply numerators and multiply denominators.

■ Division

Multiply the first fraction by the reciprocal of the second fraction.

ORDER OF OPERATIONS

Simplify within parentheses, brackets, or absolute value bars or above and below fraction bars first, in the following order.

- Apply all exponents.
- Perform any multiplications or divisions from left to right.
- Perform any additions or subtractions from left to right.

VARIABLES, EXPRESSIONS, AND EQUATIONS

An expression containing a variable is evaluated by substituting a given number for the variable.

Values for a variable that make an equation true are solutions of the equation.

REAL NUMBERS AND THE NUMBER LINE

a is less than b if a is to the left of b on the number line.

The additive inverse of x is $-x$.

The absolute value of x , denoted $|x|$, is the distance (a positive number) between x and 0 on the number line.

OPERATIONS ON REAL NUMBERS

■ Adding Real Numbers

To add two numbers with the same sign, add their absolute values. The sum has the same sign as each of the numbers being added.

To add two numbers with different signs, subtract their absolute values. The sum has the sign of the number with the larger absolute value.

Definition of Subtraction

$$x - y = x + (-y)$$

■ Subtracting Real Numbers

- Change the subtraction symbol to the addition symbol.
- Change the sign of the number being subtracted.
- Add using the rules for adding real numbers.

■ Multiplying Real Numbers

- Multiply the absolute value of the two numbers.
- If the two numbers have the same sign, the product is *positive*. If the two numbers have different signs, the product is *negative*.

Definition of Division: $\frac{x}{y} = x \cdot \frac{1}{y}$, $y \neq 0$

Division by 0 is undefined.

■ Dividing Real Numbers

- Divide the absolute value of the numbers.
- If the signs are the same, the answer is *positive*. If the signs are different, the answer is *negative*.

PROPERTIES OF REAL NUMBERS

■ Commutative Properties

$$a + b = b + a$$

$$ab = ba$$

■ Associative Properties

$$(a + b) + c = a + (b + c)$$

$$(ab)c = a(bc)$$

■ Distributive Properties

$$a(b + c) = ab + ac$$

$$(b + c)a = ba + ca$$

■ Identity Properties

$$a + 0 = a \quad 0 + a = a$$

$$a \cdot 1 = a \quad 1 \cdot a = a$$

■ Inverse Properties

$$a + (-a) = 0 \quad (-a) + a = 0$$

$$a \cdot \frac{1}{a} = 1 \quad \frac{1}{a} \cdot a = 1 \quad (a \neq 0)$$

■ Simplifying Algebraic Expressions

When adding or subtracting algebraic expressions, *only like terms* can be combined.

Linear Equations

■ Properties

- Addition:** The same quantity may be added to (or subtracted from) each side of an equality without changing the solution.
- Multiplication:** Each side of an equality may be multiplied (or divided) by the same nonzero number without changing the solution.

■ Solving Linear Equations

- Simplify each side separately.
- Isolate the variable term on one side.
- Isolate the variable.

APPLICATIONS

- Assign a variable to the unknown quantity in the problem.
- Write an equation involving the unknown.
- Solve the equation.

FORMULAS

- To find the value of one of the variables in a formula, given values for the others, substitute the known values into the formula.
- To solve a formula for one of the variables, isolate that variable by treating the other variables as constants (numbers) and using the steps for solving equations.

Exponents

For any integers m and n , the following rules hold:

■ Product Rule

$$a^m \cdot a^n = a^{m+n}$$

■ Power Rules

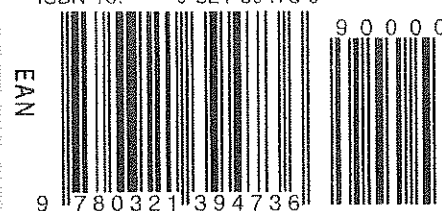
i. $(a^m)^n = a^{mn}$

ii. $(ab)^m = a^m b^m$

iii. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, $b \neq 0$

more

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Exponents (continued)

Quotient Rules

If $a \neq 0$,

- Zero exponent: $a^0 = 1$
- Negative exponents: $a^{-n} = \frac{1}{a^n}$
- Quotient rule: $\frac{a^m}{a^n} = a^{m-n}$
- Negative to positive:
 $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}, a \neq 0, b \neq 0$
 $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m, a \neq 0, b \neq 0$

Scientific Notation

A number written in scientific notation is in the form $a \times 10^n$, where a has one digit in front of the decimal point and that digit is nonzero. To write a number in scientific notation, move the decimal point to follow the first nonzero digit. If the decimal point has been moved n places to the left, the exponent on 10 is n . If the decimal point has been moved n places to the right, the exponent on 10 is $-n$.

Polynomials

A **polynomial** is an algebraic expression made up of a term or a finite sum of terms with real or complex coefficients and whole number exponents.

The **degree of a term** is the sum of the exponents on the variables. The **degree of a polynomial** is the highest degree amongst all of its terms.

A **monomial** is a polynomial with only *one* term.

A **binomial** is a polynomial with exactly *two* terms.

A **trinomial** is a polynomial with exactly *three* terms.

OPERATIONS ON POLYNOMIALS

Adding Polynomials

Add like terms.

Subtracting Polynomials

Change the sign of the terms in the second polynomial and add to the first polynomial.

Multiplying Polynomials

- Multiply each term of the first polynomial by each term of the second polynomial.
- Collect like terms.

more

Polynomials (continued)

FOIL Expansion for Multiplying Two Binomials

- Multiply the first terms.
- Multiply the outer terms.
- Multiply the inner terms.
- Multiply the last terms.
- Collect like terms.

SPECIAL PRODUCTS

Square of a Binomial

$$(x + y)^2 = x^2 + 2xy + y^2$$
$$(x - y)^2 = x^2 - 2xy + y^2$$

Product of the Sum and Difference of Two Terms

$$(x + y)(x - y) = x^2 - y^2$$

Dividing a Polynomial by a Monomial

Divide each term of the polynomial by the monomial:

$$\frac{p + q}{r} = \frac{p}{r} + \frac{q}{r}$$

Dividing a Polynomial by a Polynomial

Use long division or synthetic division.

Graphing Simple Polynomials

- Determine several points (ordered pairs) satisfying the polynomial equation.
- Plot the points.
- Connect the points with a smooth curve.

Factoring

Finding the Greatest Common Factor (GCF)

- Include the largest numerical factor of each term.
- Include each variable that is a factor of every term raised to the smallest exponent that appears in a term.

Factoring by Grouping

- Group the terms.
- Factor out the greatest common factor in each group.
- Factor a common binomial factor from the result of step ii.
- Try various groupings, if necessary.

Factoring Trinomials, Leading Term = x^2

To factor $x^2 + bx + c$, $a \neq 1$:

- Find m and n such that $mn = c$ and $m + n = b$.
- Then $x^2 + bx + c = (x + m)(x + n)$.
- Verify by using FOIL expansion.

Factoring (continued)

Factoring Trinomials, Leading Term $\neq x^2$

To factor $ax^2 + bx + c$, $a \neq 1$:

By Grouping

- Find m and n such that $mn = ac$ and $m + n = b$.
- Then $ax^2 + bx + c = ax^2 + mx + nx + c$.
- Group the first two terms and the last two terms.
- Follow the steps for factoring by grouping.

By Trial and Error

- Factor a as pq and c as mn .
- For each such factorization, form the product $(px + m)(qx + n)$ and expand using FOIL.
- Stop when the expansion matches the original trinomial.

Remainder Theorem

If the polynomial $P(x)$ is divided by $x - a$, then the remainder is equal to $P(a)$.

Factor Theorem

For a polynomial $P(x)$ and number a , if $P(a) = 0$, then $x - a$ is a factor of $P(x)$.

SPECIAL FACTORIZATIONS

Difference of Squares

$$x^2 - y^2 = (x + y)(x - y)$$

Perfect Square Trinomials

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$

Difference of Cubes

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Sum of Cubes

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

SOLVING QUADRATIC EQUATIONS BY FACTORING

Zero-Factor Property

If $ab = 0$, then $a = 0$ or $b = 0$.

Solving Quadratic Equations

- Write in standard form:
 $ax^2 + bx + c = 0$
- Factor.
- Use the zero-factor property to set each factor to zero.
- Solve each resulting equation to find each solution.

Rational Expressions

To find the value(s) for which a rational expression is undefined, set the denominator equal to 0 and solve the resulting equation.

Lowest Terms

To write a rational expression in lowest terms:

- Factor the numerator and denominator.
- Divide out common factors.

OPERATIONS ON RATIONAL EXPRESSIONS

Multiplying Rational Expressions

- Multiply numerators and multiply denominators.
- Factor numerators and denominators.
- Write expression in lowest terms.

Dividing Rational Expressions

- Multiply the first rational expression by the reciprocal of the second rational expression.
- Multiply numerators and multiply denominators.
- Factor numerators and denominators.
- Write expression in lowest terms.

Finding the Least Common Denominator (LCD)

Factor each denominator into prime factors.

- List each different factor the greatest number of times it appears in any one denominator.
- Multiply the factors from step ii.

Writing a Rational Expression with a Specified Denominator

- Factor both denominators.
- Determine what factors the given denominator must be multiplied by to equal the one given.
- Multiply the rational expression by that factor divided by itself.

Adding or Subtracting Rational Expressions

- Find the LCD.
- Rewrite each rational expression with the LCD as denominator.
- If adding, add the numerators to get the numerator of the sum. If subtracting, subtract the second numerator from the first numerator to get the difference. The LCD is the denominator of the sum.
- Write expression in lowest terms.

more>

Rational Expressions

(continued)

SIMPLIFYING COMPLEX FRACTIONS

Method 1

- Simplify the numerator and denominator separately.
- Divide by multiplying the simplified numerator by the reciprocal of the simplified denominator.

Method 2

- Multiply the numerator and denominator of the complex fraction by the LCD of all the denominators in the complex fraction.
- Write in lowest terms.

SOLVING EQUATIONS WITH RATIONAL EXPRESSIONS

- Find the LCD of all denominators in the equation.
- Multiply each side of the equation by the LCD.
- Solve the resulting equation.
- Check that the resulting solutions satisfy the original equation.

Equations of Lines Two Variables

An ordered pair is a solution of an equation if it satisfies the equation.

If the value of either variable in an equation is given, the value of the other variable can be found by substitution.

GRAPHING LINEAR EQUATIONS

To graph a linear equation:

- Find at least two ordered pairs that satisfy the equation.
- Plot the corresponding points. (An ordered pair (a, b) is plotted by starting at the origin, moving a units along the x -axis and then b units along the y -axis.)
- Draw a straight line through the points.

Special Graphs

$x = a$ is a vertical line through the point $(a, 0)$.

$y = b$ is a horizontal line through the point (a, b) .

The graph of $Ax + By = 0$ goes through the origin. Find and plot another point that satisfies the equation, and then draw the line through the two points.

more>

Equations of Lines Two Variables (continued)

Intercepts

To find the x -intercept, let $y = 0$.

To find the y -intercept, let $x = 0$.

Slope

Suppose (x_1, y_1) and (x_2, y_2) are two different points on a line. If $x_1 \neq x_2$, then the slope is

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope of a vertical line is undefined.

The slope of a horizontal line is 0.

Parallel lines have the same slope.

Perpendicular lines have slopes that are negative reciprocals of each other.

EQUATIONS OF LINES

Slope-intercept form: $y = mx + b$,

where m is the slope, and $(0, b)$ is the y -intercept.

Intercept form: $\frac{x}{a} + \frac{y}{b} = 1$,

where $(a, 0)$ is the x -intercept, and $(0, b)$ is the y -intercept.

Point-slope form: $y - y_1 = m(x - x_1)$,

where m is the slope and (x_1, y_1) is any point on the line.

Standard form: $Ax + By = C$

Vertical line: $x = a$

Horizontal line: $y = b$

Systems of Linear Equations

TWO VARIABLES

An ordered pair is a solution of a system if it satisfies all the equations at the same time.

Graphing Method

- Graph each equation of the system on the same axes.
- Find the coordinates of the point of intersection.
- Verify that the point satisfies all the equations.

Substitution Method

- Solve one equation for either variable.
- Substitute that variable into the other equation.
- Solve the equation from step ii.
- Substitute the result from step iii into the equation from step i to find the remaining value.

more>

Algebra Review

Systems of Linear Equations (continued)

Elimination Method

- Write the equations in standard form:
 $Ax + By = C$.
- Multiply one or both equations by appropriate numbers so that the sum of the coefficient of one variable is 0.
- Add the equations to eliminate one of the variables.
- Solve the equation that results from step iii.
- Substitute the solution from step iv into either of the original equations to find the value of the remaining variable.

Notes: If the result of step iii is a false statement, the graphs are parallel lines and there is no solution.

If the result of step iii is a true statement, such as $0 = 0$, the graphs are the same line, and the solution is every ordered pair on either line (of which there are infinitely many).

THREE VARIABLES

- Use the elimination method to eliminate any variable from any two of the original equations.
- Eliminate the *same* variable from any *other* two equations.
- Steps i and ii produce a system of two equations in two variables. Use the elimination method for two-variable systems to solve for the two variables.
- Substitute the values from step iii into any of the original equations to find the value of the remaining variable.

APPLICATIONS

- Assign variables to the unknown quantities in the problem.
- Write a system of equations that relates the unknowns.
- Solve the system.

MATRIX ROW OPERATIONS

- Any two rows of the matrix may be interchanged.
- All the elements in any row may be multiplied by any nonzero real number.
- Any row may be modified by adding to the elements of the row the product of a real number and the elements of another row.

A system of equations can be represented by a matrix and solved by matrix methods. Write an augmented matrix and use row operations to reduce the matrix to row echelon form.

Inequalities and Absolute Value: One Variable

Properties

- Addition:** The same quantity may be added to (or subtracted from) each side of an inequality without changing the solution.
- Multiplication by positive numbers:** Each side of an inequality may be multiplied (or divided) by the same positive number without changing the solution.
- Multiplication by negative numbers:** If each side of an inequality is multiplied (or divided) by the same negative number, the direction of the inequality symbol is reversed.

Solving Linear Inequalities

- Simplify each side separately.
- Isolate the variable term on one side.
- Isolate the variable. (Reverse the inequality symbol when multiplying or dividing by a negative number.)

Solving Compound Inequalities

- Solve each inequality in the compound inequality individually.
- If the inequalities are joined with *and*, then the solution set is the **intersection** of the two individual solution sets.
- If the inequalities are joined with *or*, then the solution set is the **union** of the two individual solution sets.

Solving Absolute Value Equations and Inequalities

Suppose k is positive.

To solve $|ax + b| = k$, solve the compound equation

$$ax + b = k \text{ or } ax + b = -k.$$

To solve $|ax + b| > k$, solve the compound inequality

$$ax + b > k \text{ or } ax + b < -k.$$

To solve $|ax + b| < k$, solve the compound inequality

$$-k < ax + b < k.$$

To solve an absolute value equation of the form $|ax + b| = |cx + d|$, solve the compound equation

$$ax + b = cx + d \text{ or}$$

$$ax + b = -(cx + d).$$

more

Inequalities and Absolute Value: One Variable (continued)

Graphing a Linear Inequality

- If the inequality sign is replaced by an equals sign, the resulting line is the equation of the boundary.
- Draw the graph of the boundary line, making the line *solid* if the inequality involves \leq or \geq or *dashed* if the inequality involves $<$ or $>$.
- Choose any point not on the line as a test point and substitute its coordinates into the inequality.
- If the test point satisfies the inequality, shade the region that includes the test point; otherwise, shade the region that does not include the test point.

Functions

Function Notation

A **function** is a set of ordered pairs (x, y) such that for each first component x , there is one and only one second component y . The set of first components is called the **domain**, and the set of second components is called the **range**.

$y = f(x)$ defines y as a function of x .

To write an equation that defines y as a function of x in function notation, solve the equation for y and replace y by $f(x)$.

To evaluate a function written in function notation for a given value of x , substitute the value wherever x appears.

Variation

If there exists some real number (constant) k such that:

$y = kx^n$, then y varies directly as x^n .

$y = \frac{k}{x^n}$, then y varies inversely as x^n .

$y = kxz$, then y varies jointly as x and z .

Operations on Functions

If $f(x)$ and $g(x)$ are functions, then the following functions are derived from f and g :

$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x) \cdot g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$$

Composition of f and g :

$$(f \circ g)(x) = f[g(x)]$$

Converting Fractions, Decimals, and Percents

Decimal

1. Divide the numerator by the denominator.
- $$\begin{array}{r} .25 \\ 4 \overline{) 1.00} \\ \underline{-8} \\ 20 \\ \underline{-20} \\ 0 \end{array}$$

Fraction

convert to

$$\frac{1}{4}$$

Percent

1. Divide the numerator by the denominator.
- $$\begin{array}{r} .25 \\ 4 \overline{) 1.00} \end{array}$$
2. Multiply by 100 or move the decimal point two places to the right.
 $0.25 \times 100 = 25.00$ or $\underline{.25} = 25$
3. Add the percent symbol. 25%

Percent

1. Multiply by 100 or move the decimal point two places to the right.
 $0.75 \times 100 = 75.00$ or $\underline{0.75} = 75$
2. Add the percent symbol.
 $0.75 = 75\%$

Decimal

convert to

$$0.75$$

Fraction

1. Use the place value of the last digit to the right of the decimal point as the denominator. *hundredths* = $\frac{75}{100}$
2. Remove the decimal point and make that number the numerator. $0.75 = \frac{75}{100}$
3. Reduce the fraction to lowest terms. $\frac{75}{100} = \frac{3}{4}$

Fraction

1. Remove the percent symbol and make that number the numerator.
2. Use 100 as the denominator. $\frac{40}{100}$
3. Reduce the fraction to lowest terms. $\frac{40}{100} = \frac{2}{5}$

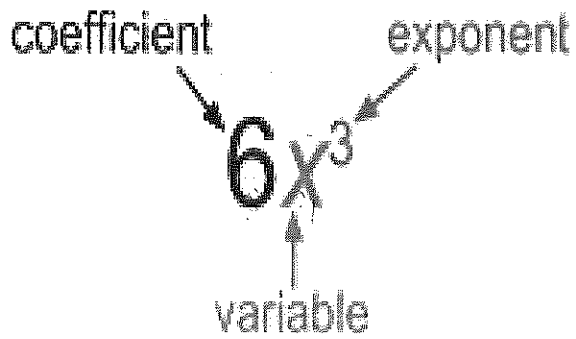
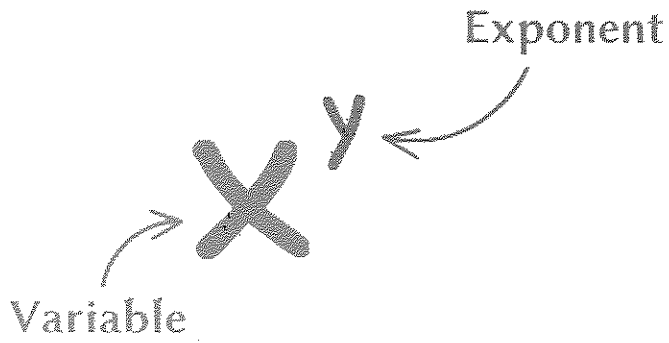
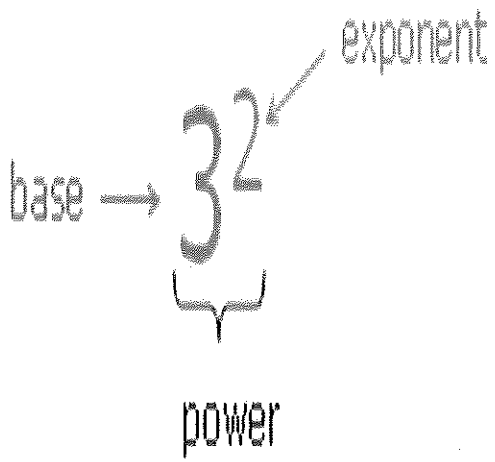
Percent

convert to

$$40\%$$

Decimal

1. Remove the percent symbol. 40
2. Divide by 100 or move the decimal point two places to the left.
 $\frac{40}{100} = 0.40$
 $40.0 = 0.40$



Addition	increased by more than combined, together total of sum, plus added to
Subtraction	decreased by minus, less difference between/of less than, fewer than
Multiplication	of times, multiplied by product of increased/decreased by a factor of (this type can involve both addition or subtraction <i>and</i> multiplication!)
Division	per, a out of ratio of, quotient of percent (divide by 100)
Equals	is, are, was, were, will be gives, yields sold for

Steps to Solve Algebra Word Problems

Equations are frequently used to solve practical problems.

The steps involved in the method of solving an algebra word problem are as follows.

STEP 1 :

Read the problem carefully and note down what is given and what is required.

STEP 2 :

Select a letter or letters say x (and y) to represent the unknown quantity(ies) asked for.

STEP 3 :

Represent the word statements of the problem in the symbolic language step by step.

STEP 4 :

Look for quantities which are equal as per conditions given and form an equation or equations.




STEP 5 :

Solve the equation(s) obtained in step 4.

STEP 6 :

Check the result for making sure that your answer satisfies the requirements of the problem.

Metric Conversion

K ing	H enry	D ied	U nusually 	D rinking	C hocolate	M ilk
Kilo  10 x 10 x 10 x LARGER than a unit 1 kilo = 1,000 units	Hecto 10 x 10 x LARGER than a unit 1 hecto = 100 units	Deca 10 x LARGER than a unit 1 deca = 10 units	* Unit * Meter (length) Liter (liquid volume) Gram (mass/weight) 1 unit	Deci 10 x SMALLER than a unit 10 deci = 1 unit	Centi 10 x 10 x SMALLER than a unit 100 centi = 1 unit	Milli 10 x 10 x 10 x SMALLER than a unit  1,000 milli = 1 unit
km = kilometer kl = kiloliter kg = kilogram	hm = hectometer hl = hectoliter hg = hectogram	dam = decameter dal = decaliter dag = decagram	m = meter L = liter g = gram	dm = decimeter dL = deciliter dg = decigram	cm = centimeter cL = centiliter cg = centigram	mm = millimeter mL = milliliter mg = milligram

Example: 5 kilo

50 hecto

500 deca

5,000 units

50,000 deci

500,000 centi

5,000,000 milli

← **DIVIDE** numbers by 10 if you are getting bigger (same as moving decimal point one space to the left)

MULTIPLY numbers by 10 if you are getting smaller (same as moving decimal point one space to the right) →

