COLLEGE PREP
SECTION 5A – GETTING READY FOR CHAPTER 5

Objectives:

- Simplify exponential equations using the product rule, the quotient rule, the power rule, and the Law of Exponents.
- Evaluate Exponential expressions with a Zero or negative exponent.
- Convert between Scientific Notation and Decimal Notation.
- Use Scientific Notation to multiply and divide.

NOTATION: in the expression a^n, a is called the ***base***, and n is called the ***exponent or power.***

Rules for Operations with Exponents

Operation	Formula	Example
Multiplying – add exponents	$a^m \cdot a^n = a^{m+n}$	$x^2 \cdot x^5 = x^7$
Dividing – subtract exponents	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{r^8}{r^3} = r^5$
Power to a power – multiply exponents	$(a^m)^n = a^{mn}$	$(y^3)^4 = y^{12}$
Power of a product – exponent applies to each factor (like distributing)	$(ab)^n = a^n b^n$	$(2c)^4 = 16c^4$
Power of a quotient – exponent applies to numerator and denominator (like distributing)	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{5}{w}\right)^3 = \frac{125}{w^3}$
Power of a negative quotient – exponent applies to numerator and denominator (like distributing) This will cause everything inside to switch places.	$\left(\frac{a}{b}\right)^{-n} = \frac{a^{-n}}{b^{-n}} = \frac{b^n}{a^n}$	$\left(\frac{x}{5}\right)^{-2} = \frac{x^{-2}}{5^{-2}} = \frac{5^2}{x^2}$
Negative exponents – moving the exponential factor to the denominator creates a positive exponent	$a^{-n} = \frac{1}{a^n}$ or $\frac{1}{a^{-n}} = a^n$	$3x^{-4} = \frac{3}{x^4}$
Zero Exponents – any number or variable that has a zero exponent is always equal to 1	$\frac{a^m}{a^m} = a^0 = 1$	$\frac{x^0}{4} = \frac{1}{4}$

Note: These power rules assume that the variable does not equal 0 whenever it's in the denominator or if it is raised to the zero power.

BEWARE of these common mistakes!!!

Error Formula	Description	Example
$(a+b)^n \neq a^n + b^n$	Exponents ***do not*** distribute over addition or subtraction!	$(3+5)^2 \neq 3^2 + 5^2$
$\frac{a+b}{a+c} \neq \frac{b}{c}$	Like terms in fractions ***do not*** cancel! (Only *factors* cancel.)	$\frac{2+5}{3+5} \neq \frac{2}{3}$
$a^0 \neq 0$	Zero exponent ***does not*** mean the same as multiply by zero!	$5^0 \neq 0$
$a^{-n} \neq -a^n$	Negative exponents ***do not*** make a monomial negative!	$4^{-3} \neq -4^3$

EXAMPLES: Simplify each expression

Product Rule: Remember to deal with the coefficients separately.

A) $3^2 \cdot 3^3 = 3^{2+3} = 3^5 = 243$

B) $2z^2 \cdot 5z^4 = 2 \cdot 5 \cdot z^{2+4} = 10z^6$

Quotient Rule: Coefficients (numbers) are divided, exponents are subtracted.

C) $\frac{6^4}{6} = 6^{4-1} = 6^3 = 216$

D) $\frac{25m^8}{15m^3} = \left(\frac{25}{15}\right) \cdot m^{8-3} = \frac{5}{3}m^5$

Zero Exponent Rule: Anything with an exponent of zero should be changed to a 1

E) $5^0 = 1$

F) $18x^0 = 18(1) = 18$

Negative Exponent Rule: Move ONLY the variable that the exponent is attached to. If it's outside parentheses, move everything within the parentheses.

G) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

H) $5b^{-4} = \frac{5}{b^4}$

I) $\left(\frac{x}{2}\right)^{-3} = \frac{x^{-3}}{2^{-3}} = \frac{2^3}{x^3} = \frac{8}{x}$

J) $\frac{5}{3}z^{-3} \cdot \left(-\frac{9}{20}z^4\right) = \left(\frac{5}{3} \cdot \frac{-9}{20}\right) \cdot z^{-3+4} = \frac{-3}{4}z$

Power Rule: If you raise a power to a power, you are multiplying it by itself, therefore, you must raise any coefficient to the power outside the parentheses, and multiply all exponents.

K) $(3^2)^4 = 3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2 = 3^{2 \cdot 4} = 3^8$

L) $(7^2)^0 = 7^{2 \cdot 0} = 7^0 = 1$

M) $(2a)^4 = 2^4a^4 = 16a^4$

N) $(-4b^3)^{-2} = (-4)^{-2} \cdot b^{3 \cdot -2} = \frac{1}{(-4)^2b^6} = \frac{1}{16b^6}$

O) $\left(\frac{z}{5}\right)^3 = \frac{z^3}{5^3} = \frac{z^3}{125}$

P) $\left(\frac{5b^3}{c^2}\right)^2 = \frac{5^2b^{3 \cdot 2}}{c^{2 \cdot 2}} = \frac{25b^6}{c^4}$

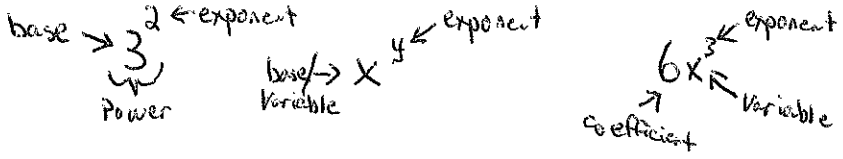
SCIENTIFIC NOTATION. A number is written in scientific notation when it is in the form $a \times 10^n$ where $1 \leq |a| \leq 10$ and n is an integer.

To change a decimal to scientific notation:

Step 1: Count the number N of decimal places that the decimal point must be moved in order to get only one digit (a) in front of the decimal.

Step 2: If you had to move the decimal to the left (you started with a large number), then your exponent is positive ($a \times 10^N$). If you had to move the decimal to the right (you started with a decimal), then your exponent will be negative ($a \times 10^{-N}$).

①



Polynomials

Laws of Exponents (There are more, but these are the ones we will be concerned with)

1. Multiplication of Powers
   - when the bases are the same, add the exponents.

ie: $(2^5)(2^4) = 2^9 = 512$

$(x^3)(x^2) = x^5$

2. Division of Powers
   - when the bases are the same, subtract the exponents

ie: $(2^5) \div (2^4) = 2^1 = 2$

$x^3/x^2 = x^1 = x$

3. Raising a Power to a Power
   - when the bases are the same, the exponents are multiplied

ie: $(5^2)^3 = (5^2)(5^2)(5^2) = 5^6$

$(x^4)^3 = (x^4)(x^4)(x^4) = x^{12}$

4. Raising a Product to a Power
   - each base within the product have the exponent

ie: $(2 \times 3)^4 = 2^4 3^4 = (16)(81) = 1296$

$(xy)^3 = x^3 y^3$

5. Raising a Quotient or Fraction to a Power
   - the numerator and denominator have the exponent

ie: $\left(\frac{3}{4}\right)^5 = \frac{3^5}{4^5} = \frac{243}{1024}$

$$\left(\frac{c}{d}\right)^3 = \frac{c^3}{d^3}$$

6. A Caution
- the five exponent laws shown involve multiplication and division ONLY and DO NOT work for addition or subtraction.

ie: $(2+3)^4 = 5^4 = 625$
but $(2+3)^4 \neq 2^4 + 3^4$ $(16 + 81 = 97)$

ie: $(c+d)^n \neq c^n + d^n$
$(c-d)^n \neq c^n - d^n$

*7. Power of a Negative Exponent (Quotient)
- numerator and denominator have the same exponent
- in math, we try to avoid negative exponents whenever possible, so we find the reciprocal (flip)

ie: $\left(\frac{3}{5}\right)^{-2} = \frac{3^{-2}}{5^{-2}} = \frac{5^2}{3^2} = \frac{25}{9}$

$$\left(\frac{a}{b}\right)^{-3} = \frac{a^{-3}}{b^{-3}} = \frac{b^3}{a^3}$$

*8. Negative Exponents
- move the exponential factor to the denominator (or numerator) (which gives us a positive exponent)

ie: $3x^{-4} = \frac{3}{x^4}$

$$a^{-n} = \frac{1}{a^n}$$

$$\frac{1}{a^{-n}} = a^n$$

9. Zero Exponents

- any number or variable that has a zero exponent is ALWAYS equal to 1

ie: $\frac{5^2}{5^2} = 5^0 = 1$

$\frac{x^3}{x^3} = x^0 = 1$

10. More Cautions

- Zero exponent DOES NOT mean the same as multiply by zero.

ie: $a^0 \neq 0$; $5^0 \neq 0$

- Negative exponents do not make the base negative

ie: $a^{-n} \neq -a^n$; $4^{-3} \neq -4^3$

11. Identity Exponent Property

- a non-zero quantity raised to the first power is equal to itself

ie: $x^1 = x$

$8^1 = 8$

Monomial

- a variable, a real number, or a multiplication of one or more variables and a real number with whole number exponents.

ie: 9 ; x ; $9x$; $6xy$; $0.60x^4y$

NOT monomials: $y-6$; x^{-1} or $1/x$; $6+x$; a/x

Binomial

- two monomials (terms) seperated by addition or subtraction signs, but NEVER multiplication or division signs

ie: $2x+5$; x^2-x ; $x-5$; $x+2^{-1}$ (NOTE: this is a binomial because the negative exponent is attached to a real number, making it a real number when simplified, not to the variable)

Trinomial

- three monomials (terms) seperated by addition or subtraction signs, but NEVER multiplication or division signs.

ie: x^2+5x+6 ; x^5-3x-8

Polynomial

- a monomial, binomial or trinomial

Evaluating Polynomials by Substitution.

- subsitute the given value for the variable. Remember BEDMAS! and Exponent Laws.

ie: $7x^2-3x+2$ for $x=-2$

$= 7(-2)^2 - 3(-2) + 2$

$= 7(4) + 6 + 2$

$= 28 + 6 + 2$

$= 36$

- do the same steps if you have two variables.

Degree of a Polynomial

- always write polynomials in alphabetical order

ie $a^2b^3c^4$ not $c^4b^3a^2$

- once in alphabetical order, we calculate the degree of each monomial by looking at the exponents of each variable in each term, then put them in descending (highest to lowest) order.

  ie: t^3 → degree 3
  x^2y^2 → degree 4
  $2x$ → degree 1

- ensure you combine like terms, and do all other calculations, before putting them in alphabetical and descending order.

Adding and Subtracting Polynomials

- follow the same rules as we learned in algebra unit (watch those negative signs!) and collect like terms.